

Sensitivity analysis of the peak outflow induced by the breaching of embankment dams

Sensitivitätsanalyse zum Breschendurchfluss bei dem Versagen eines Staudamms

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Abstract

Accurately predicting the flow induced by the collapse of embankment dams remains challenging, as a consequence of the level of uncertainty affecting the prediction of breach width and formation time. Moreover, very few systematic analyses are available today concerning the sensitivity of the flow with respect to those uncertainties. The present theoretical analysis quantifies the influence of variations in the breach parameters on the peak outflow.

Zusammenfassung

Genaue Prognosen über den Breschendurchfluss infolge des Versagens eines Staudamms gestalten sich schwierig, weil Annahmen über den zeitlichen Verlauf der Breschenbildung sowie über die Breschenbreite mit Unsicherheiten behaftet sind. Außerdem sind nur wenige systematische Analysen zur Sensitivität des Abflusses in Bezug auf diese Unsicherheiten verfügbar. Die theoretische Analyse, die hier dargestellt wird, bestimmt quantitativ den Einfluss von Schwankungen in den Bruchparametern auf die Abflussspitze.

1 Introduction

Among all existing dams in the world, embankment dams constitute the vast majority of them and the past experience emphasizes the fact that they play an important part in numerous dam accidents. Hence, the risk associated to these embankment dams, even of medium or small sizes, may not be neglected. However, accurately simulating the flood wave induced by the breaching of embankment dams remains a challenging task, especially in the near field, as a result of the significant uncertainties affecting the evaluation of the breach parameters, such as the final width and the formation time.

Those breach parameters are usually evaluated by means of prediction equations (e.g. Froehlich [1]), leading to a most likely value for the main breach parameters, but no direct quantification of uncertainty although this uncertainty is widely recognized as particularly large, as confirmed quantitatively by Wahl [2]. Besides, very few systematic analysis are available today concerning the sensitivity of the induced flood wave with respect to the uncertainty in the breach parameters, such as done for instance by Singh and Snorrason [3].

To contribute to filling this lack of knowledge, a theoretical analysis has been developed to quantify the effect that uncertainty on breach parameters may have on the hydraulic results, namely the outflow hydrograph at the breach and, in particular, the peak discharge. Therefore

an approximated (linearized) formulation of the hydraulic model is derived (para. 2), for which an exact analytical solution is sought (para. 3). This solution enables to directly compute the derivative of the outflow discharge with respect to the breach parameters, and thus to quantitatively evaluate its sensitivity, based on the concept of “relative variation rate” (para. 4). Finally, the conclusions drawn from the theoretical developments are illustrated and validated by comparison with a real example (para. 5).

2 Simplified hydraulic model

The simplified approach exploited for describing the flow in the reservoir and through the breach is a lumped model based on the continuity equation, expressing the volume conservation in the reservoir (**Figure 1**):

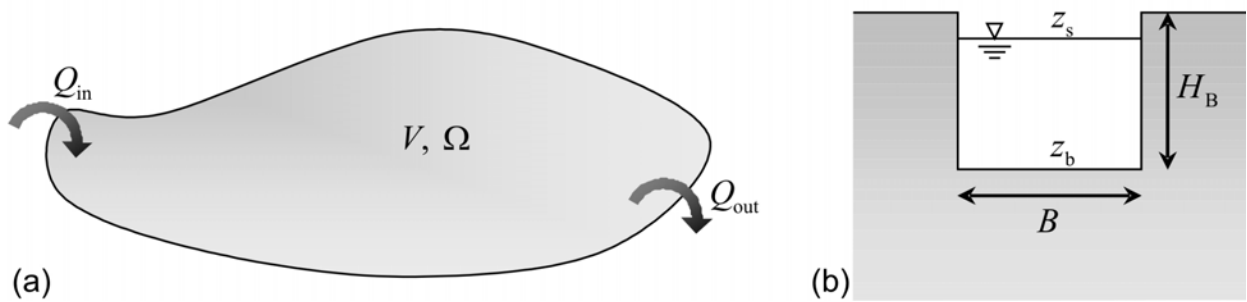


Figure 1: Sketch of the reservoir (a) and of the idealized breach (b), with main notations.

$$\frac{dV}{dt} = Q_{in} - Q_{out} \quad \Leftrightarrow \quad \Omega \frac{dz_s}{dt} = Q_{in} - Q_{out}, \quad (1)$$

where V (m^3) represents the volume of water stored in the reservoir, while Q_{in} (m^3/s) and Q_{out} (m^3/s) designate respectively the inflow and outflow discharges. Eq. (1) also shows that the conservation of the volume of water can alternatively be expressed as a function of the water level z_s , with Ω (m^2) representing the surface of the reservoir.

In the present case, Q_{in} is assumed to be zero. The most simple and widespread approach to evaluate the discharge Q_{out} through the breach consists in exploiting Poleni’s formula, normally valid for weirs. Such an approach is obviously sufficient for the present theoretical analysis, since the model will be further simplified in the sake of finding an analytical solution. In contrast, enhanced formulas can also be applied to account for the more complex real flow prevailing through the breach and for possible backwater effects (e.g. Kamrath *et al.* [3]).

Since the outflow discharge is easily expressed as a function of the head H above the breach crest, the continuity equation (1) is advantageously formulated in terms of this variable as main unknown. For this purpose, provided that velocities in the reservoir may be neglected, the following change of variable can be used: $H = z_s - z_b$, with z_b designating the level of the breach crest, leading to the following reformulation of Eq. (1) :

$$\frac{dH}{dt} + \frac{Q_{out}}{\Omega} = - \frac{dz_b}{dt}. \quad (2)$$

In order to obtain a fully analytical solution and to facilitate the analysis and interpretation, the following additional assumptions are introduced. First, the reservoir is considered as prismatic (Ω constant). Second, a rectangular shape is assumed for the breach (B constant). Third, the time evolution of the level z_b of the breach crest is approximated by a linear function of time. Consequently, the time derivative of z_b is simply given by: $dz_b/dt = \varepsilon(-H_b/T_f)$, where H_b represents the final breach height, T_f designates the breach formation time and the factor ε is defined as follows: $\varepsilon = 1$ for $t < T_f$ and $\varepsilon = 0$ for $t > T_f$. Finally, a linear relationship between the outflow discharge and the head H is adopted to enable a fully analytical solution. The outflow discharge is thus approximated by:

$$Q_{\text{out}} = B\mu H, \quad (3)$$

where the *discharge coefficient* μ is assumed to remain constant. As a result, Eq. (2) takes the form of the following linear differential equation:

$$\frac{dH}{dt} + \frac{B\mu}{\Omega} H = \varepsilon \frac{H_b}{T_f}. \quad (4)$$

3 Analytical solution

Eq. (4) admits the following exact analytical solution:

$$H(t) = H_0 e^{-\alpha(t-t_0)} + \varepsilon \frac{H_b}{\alpha T_f} (1 - e^{-\alpha(t-t_0)}) \quad \text{with} \quad \alpha = \frac{B\mu}{\Omega}, \quad (5)$$

where H_0 represents the head at time $t = t_0$.

For the first phase (breach formation), $H_0 = 0$ at $t_0 = 0$. Consequently, Eq. (5) becomes, for $0 < t < T_f$:

$$H_1(t) = \frac{H_b}{\alpha T_f} (1 - e^{-\alpha t}). \quad (6)$$

During this first phase, the outflow discharge continuously increases. The time derivative of H_1 remains positive in the whole interval $0 < t < T_f$ and its maximal value $H_f = H_1(T_f)$ is reached at $t = T_f$.

The second phase (reservoir drawdown after complete breach formation) starts at $t_0 = T_f$, with $H_0 = H_f$, so that Eq. (5) becomes:

$$H_2(t) = \frac{H_b}{\alpha T_f} (1 - e^{-\alpha T_f}) e^{-\alpha(t-T_f)}. \quad (7)$$

This expression is however a decreasing function of time. Therefore, according to the present simple model, the peak outflow Q^* (m^3/s) is reached at $t = T_f$:

$$Q^* \square Q_{\text{out}}(T_f) = B\mu H_f = \frac{\Omega H_b}{T_f} \left(1 - e^{-\frac{B\mu T_f}{\Omega}} \right). \quad (8)$$

4 Sensitivity of the peak discharge

The relative variation rate R_β (-) of the peak discharge Q^* with respect to any parameter β can be defined as:

$$R_\beta \square \frac{dQ^*}{d\beta} \left(\frac{Q^*}{\beta} \right)^{-1}, \quad (9)$$

which is a non-dimensional expression whatever the dimensions of the parameter β . If $|R_\beta|$ takes values higher than one, a variation (uncertainty) in the parameter β induces a variation (uncertainty) in the peak discharge which is more than proportional; and vice-versa if $|R_\beta|$ is lower than one.

4.1 Influence of the breach formation time

The relative variation rate of the peak outflow with respect to the breach formation time is evaluated by :

$$R_{T_f} = \frac{dQ^*}{dT_f} \left(\frac{Q^*}{T_f} \right)^{-1} = \frac{(\tau + 1)e^{-\tau} - 1}{1 - e^{-\tau}}. \quad (10)$$

where the non-dimensional parameter τ has been introduced and is defined as:

$$\tau \square \alpha T_f = \frac{B\mu T_f}{\Omega} \quad \Rightarrow \quad \tau = \frac{B\mu H_b}{\Omega} \left(\frac{H_b}{T_f} \right)^{-1} \square \frac{Q_{\text{out}}/\Omega}{H_b/T_f}. \quad (11)$$

It can be interpreted as a non-dimensional expression of the breach formation time or as the ratio between the rates of variation in the head H , induced, on one hand, by the drawdown of the reservoir (as a result of the outflow discharge) and, on the other hand, by the decrease of the breach crest level.

As already suggested in the solution expressed by Eq. (6), if τ has a high value (wide breach, small reservoir), the head in the reservoir declines quickly during the formation of the breach. On the contrary, if τ remains smaller, the head in the reservoir is hardly modified during the breach formation. Consequently, for a low value of τ , the sensitivity of the released discharged with respect to the formation time is particularly small. Indeed, whatever the formation time, the major part of the initial head in the reservoir remains available until the end of the breach formation and thus the peak discharge occurs at this time, with z_s only slightly reduced compared to its initial value. Conversely, a higher parameter τ indicates a much higher sensitivity of the peak discharge with respect to the formation time.

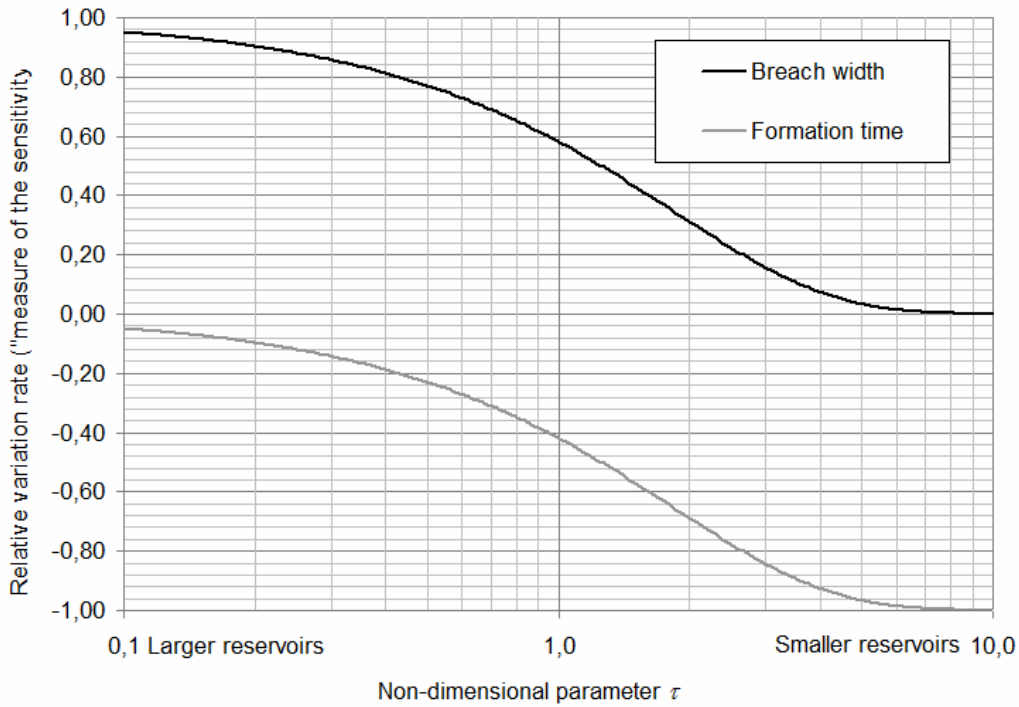


Figure 2: Relative variation rate of the peak outflow at the breach as a function of the non-dimensional parameter τ .

These conclusions are corroborated by the curve in **Figure 2**, which plots the right hand side of Eq. (10) as a function of the parameter τ . This relative rate of variation is always negative, reflecting that an increase in the breach formation time necessarily reduces the peak discharge. The absolute value of the relative rate of variation is small for low values of τ and rises for higher values of τ , consistently with the interpretation above.

4.2 Influence of the breach width

An expression similar to Eq. (10) can be derived to assess the sensitivity of the peak discharge with respect to the breach width:

$$R_B = \frac{dQ^*}{dB} \left(\frac{Q^*}{B} \right)^{-1} = \frac{\tau e^{-\tau}}{1 - e^{-\tau}}. \quad (12)$$

This expression is also plotted in Figure 2. In contrast with the result obtained for the breach formation time, the influence of the breach width is clearly higher for larger reservoirs (larger Ω , i.e. small τ) and vice-versa for smaller reservoirs (smaller Ω , i.e. larger τ). Indeed, in the case of a very large reservoir, for which the water level changes little during the formation of the breach, a variation in the width of the (rectangular) breach logically provokes an almost proportional modification in the outflow discharge. On the contrary, if the reservoir is much smaller, an increase in the breach width generates a quicker decrease in the water level upstream and results thus in a significantly less than proportional increase in the peak discharge.

5 Application

For validation purpose, the theoretical developments above have been applied to predict the sensitivity of the outflow with respect to the breach formation time in the case of the breaching of a real 20 m-high rockfill embankment dam (crest length: 250 m, reservoir capacity: $17 \cdot 10^6 \text{ m}^3$).

To provide reference data for validation purpose, the induced flow has been computed with the two-dimensional hydrodynamic model WOLF 2D. This 2D model solves the shallow-water equations by a finite volume method. Flux evaluation is based on an original flux-vector splitting technique [5, 6]. WOLF 2D has been extensively validated by comparisons with experimental data and field measurements (for instance, benchmarks from EU Projects such as CADAM and IMPACT have been tested successfully [6]).

Figure 3 displays the outflow hydrograph computed by the 2D hydrodynamic model for three different breach formation times. The values of the corresponding peak discharges can be used directly to evaluate the actual relative variation rate using the definition given by Eq. (9), with β standing for the breach formation time: $R_{T_f} \approx -0.5$ (averaged value).

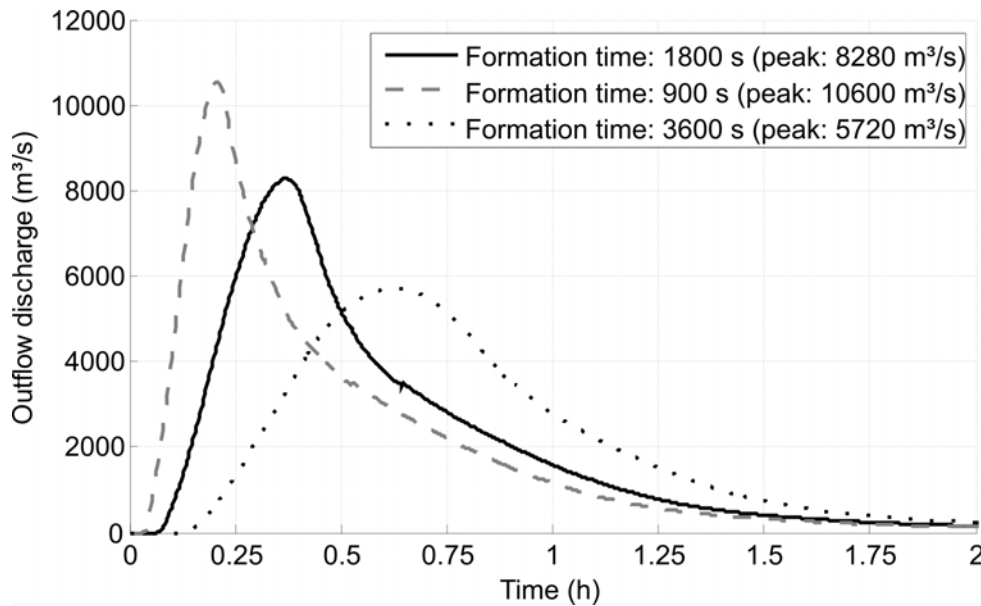


Figure 3: Outflow hydrograph (m^3/s) computed by the 2D hydrodynamic model for three different breach formation times.

Eq. (10) can also be applied to predict the value of R_{T_f} . The parameter τ is computed from the following orders of magnitude: $B \approx 200 \text{ m}$, $\mu \approx 5$, $T_f \approx 1800 \text{ s}$ and $\Omega \approx 1,65 \cdot 10^6 \text{ m}^2$, which leads to $\tau \approx 1,1$ and, deduced from Eq. (10), $R_{T_f} \approx -0.45$. This prediction compares reasonably well with the reference value obtained from the 2D numerical simulation.

6 Conclusion

A lumped hydraulic model has been derived, in which the descriptions of hydrodynamics and geometry are simplified in such a way that the solution can be written out analytically. Based on

this analytical solution, the sensitivity of the peak discharge has been quantified non-dimensionally thanks to the concept of relative variation rate.

A simple analytical relation has been obtained to evaluate this relative variation rate as a function of a single non-dimensional parameter characterizing globally the reservoir and the breach. For larger reservoirs (resp. narrower breach), the peak discharge is shown to be extremely sensitive to the breach width and almost not influenced by the breach formation time. In contrast, for smaller reservoirs (resp. wider breach), the outflow is less affected by the breach width but is strongly influenced by the breach formation time.

For practical applications those results help to identify the most influential breach parameters and consequently to directly devote a maximum of resources on a better estimation of them. The applicability of those results has been highlighted through one example involving the breaching of a real 20 m-high rockfill dam.

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